Module 4

M4.Behaviour of Laminae - II

Learning Units of Module 4

M4.1 Micromechanics of Laminae

M4.2 Micromechanics of Laminae

Note:

- So far we have talked on apparent homogenized properties of a fiber reinforced lamina.
- Now we will examine how we can calculate the homogeneous lamina properties from the heterogeneous composite material constituent properties

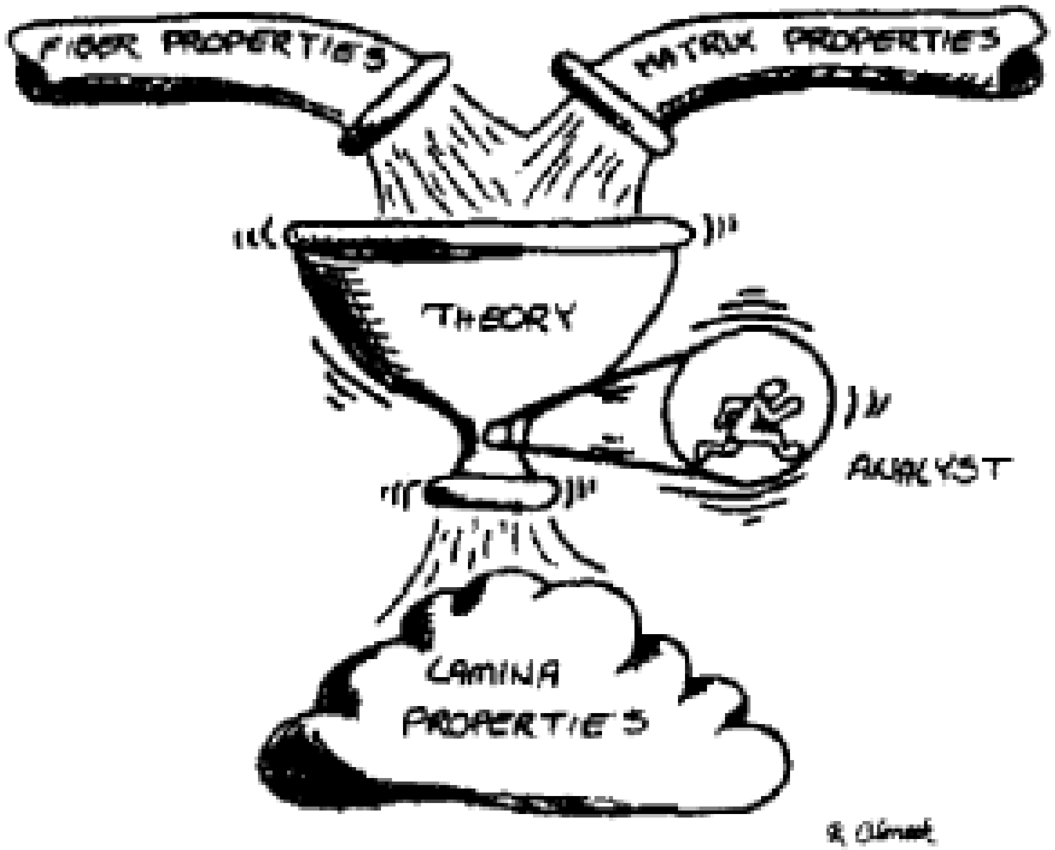
Definition of Micromechanics

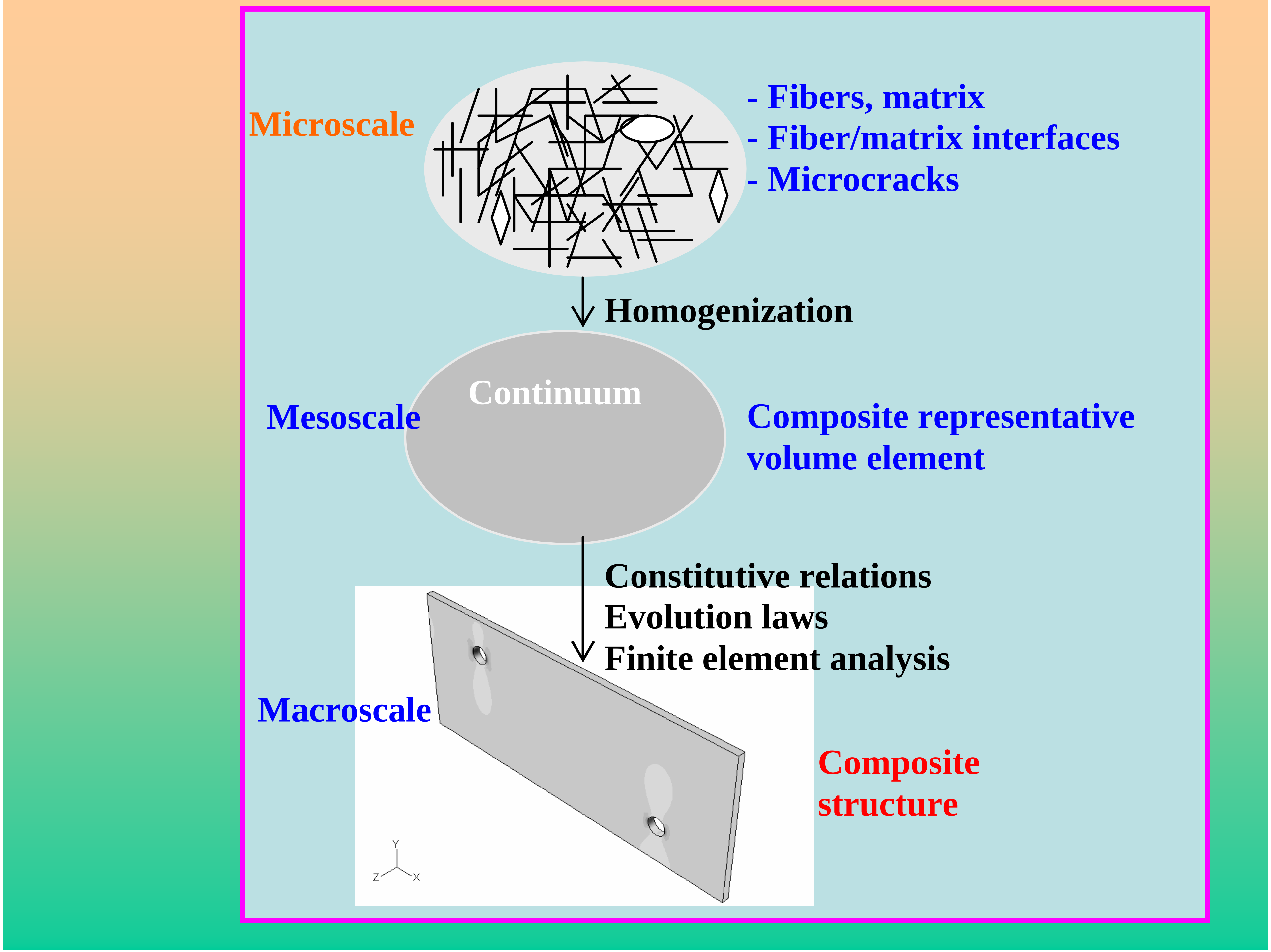
Definition

- The term "micromechanics" does not refer to mechanical behavior at the molecular level. Looks at components of a composite, the matrix and the fiber, and tries to predict the behavior of the assumed homogeneous composite material. The behavior or the lamina is called "macro-mechanics".
- Study of mechanical behavior of a composite material in terms of its constituent materials"

Micromechanics

Determining unknown properties of the composite based on known properties of the fiber and matrix




Microscale
- Fibers, matrix
- Fiber/matrix interfaces
- Microcracks
Homogenization
Continuum
Mesoscale
Composite representative
volume element
Constitutive relations
Evolution laws
Finite element analysis
Macroscale
Composite
structure
Y
Z
X

Uses of Micromechanics

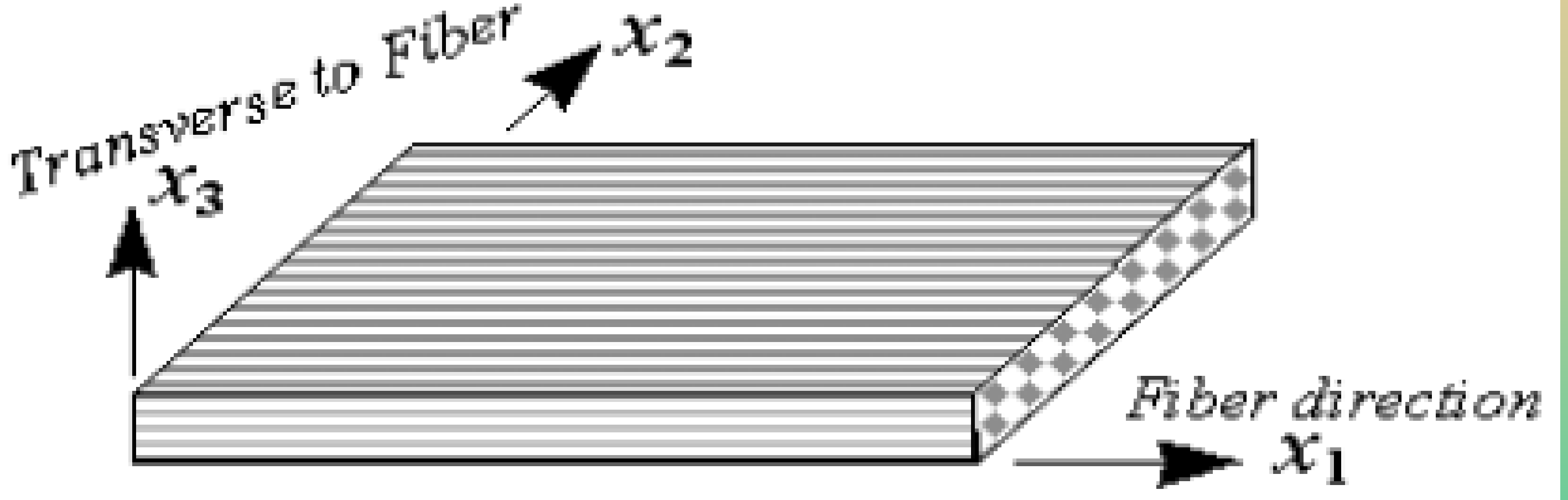
- Predict composite properties from fiber and matrix data.
- Extrapolate existing composite property data to different fiber volume fraction or void content.
- Check experimental data for errors.
- Determine required fiber and matrix properties to produce a desired composite material.
- The relation between ply uniaxial strengths and constituent properties of structure is obtained through the composite **micromechanics**.

Limitations of Micromechanics

- Predicted composite properties are only as good as fiber and matrix properties used.
- Simple theories assume isotropic fibers – many fiber reinforcements are orthotropic.
- Some properties are not predicted well by simple theories more accurate analyses are time consuming and expensive.
- Predicted strengths are upper bounds

Definition of Lamina

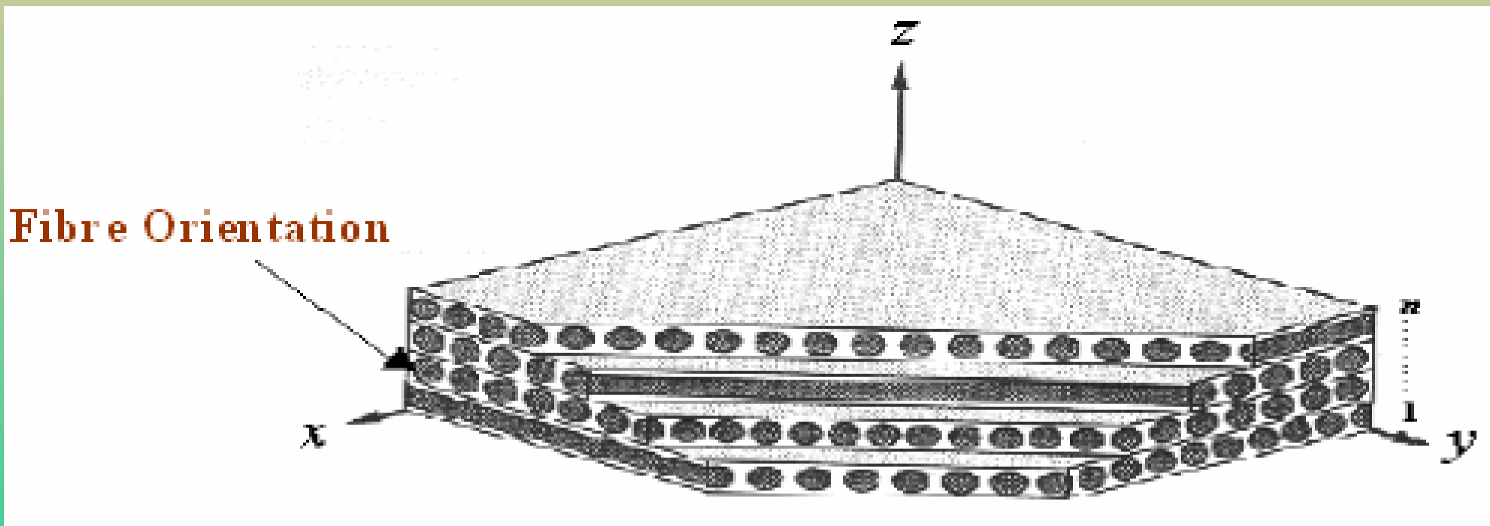
- **Lamina (or Ply):** It is a single layer (plane or curved) of unidirectional or woven fabric in a matrix.



Uni-directional Laminate

Definition of Laminate

- **Laminate:** Two or more unidirectional laminae or a ply stacked together at various orientations is called Laminate.
  The laminae (plies) can be of various thicknesses and consists of different or same materials. Ex: (0/90/0), (0/0/0/0) = (04)



Laminate Stacking Sequence

Continued……

Hybrid Composites: A composite made up of 2 or more different types of materials. Ex: $(0^k/0^k/45^c/90^c)$

- **Inter-ply Hybrid Composites:** Laminate made up of plies of Glass/Epoxy, Carbon/Epoxy & Aramid/Epoxy.
- **Intra-ply Hybrid Composites:** A ply made up of two or more fibers intermingled.
- **Intra-Interplay Hybrid Composites:** Combination of Intraply and Interply hybrid.
- **Lay-up:** Configuration of a laminate that defines ply composition. (0/±45/90)
- **Stacking sequence:** Configuration of a laminate that defines ply composition and the exact sequence of ply orientation and its thickness.

Different Types Laminate Sequences

Example	Laminate Stacking Sequence
Unidirectional:	$[0/0/0/0/0/0] = [0_6]$
Cross-ply symmetric:	$[0/90/90/0] = [0/90]_s$
Angle-ply symmetric:	$[45/-45/-45/45] = [45/-45]_s = [\pm 45]_s$
Angle-ply asymmetric:	$[30/-30/30/-30/30/-30/30/-30] = [30/-30]_4 = [\pm 30]_4$
Quasi isotropic:	$[0/+45/-45/90]$
Symmetric Quasi-isotropic:	$[0/+45/-45/90/90/-45/45/0] = [0/\pm 45/90]_s$
Multidirectional:	$[0/+45/30/-30/45]$
Hybrid:	$[0^k/0^k/45^c/-45^c/90^G/-45^c/45^c/0^k/0^k]_T = [0_2^k/\pm 45^c/90^G]_s$

Define: S=Symmetric, K=Kevlar, C=Carbon, G=Glass, T=Total

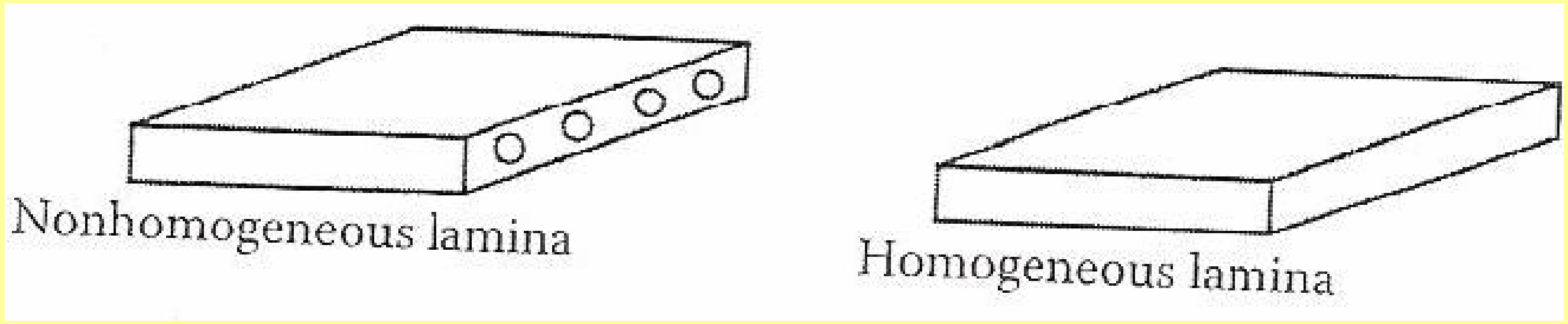
Methods Of Micromechanics

1. Mechanics of Materials
2. Elasticity Approach:
   - (i) Boundary principles
   - (ii) Exact solutions
   - (iii) Approximate solutions
3. Numerical :
   - (i) Finite-difference,
   - (ii) Finite element,
   - (iii) Boundary element method
4. Experimental: Photoelasticity

In this course we address only the simple "Mechanics of materials" method.

Micromechanics and Assumptions

- Approach: mechanics of materials approach, semi-empirical
- Assumption: the lamina is looked at as a material whose properties are different in various directions, but not different from one location to another.



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The Lamina is :	Macroscopically homogeneous Linearly elastic Macroscopically Orthotropic Initially stress free
The fibers are :	Homogeneous Linearly elastic Isotropic/Orthotropic Regularly spaced Perfectly aligned
The matrix is :	Homogeneous Linearly elastic Isotropic

Evaluation of Four Elastic Moduli

There are four elastic moduli of a unidirectional lamina:

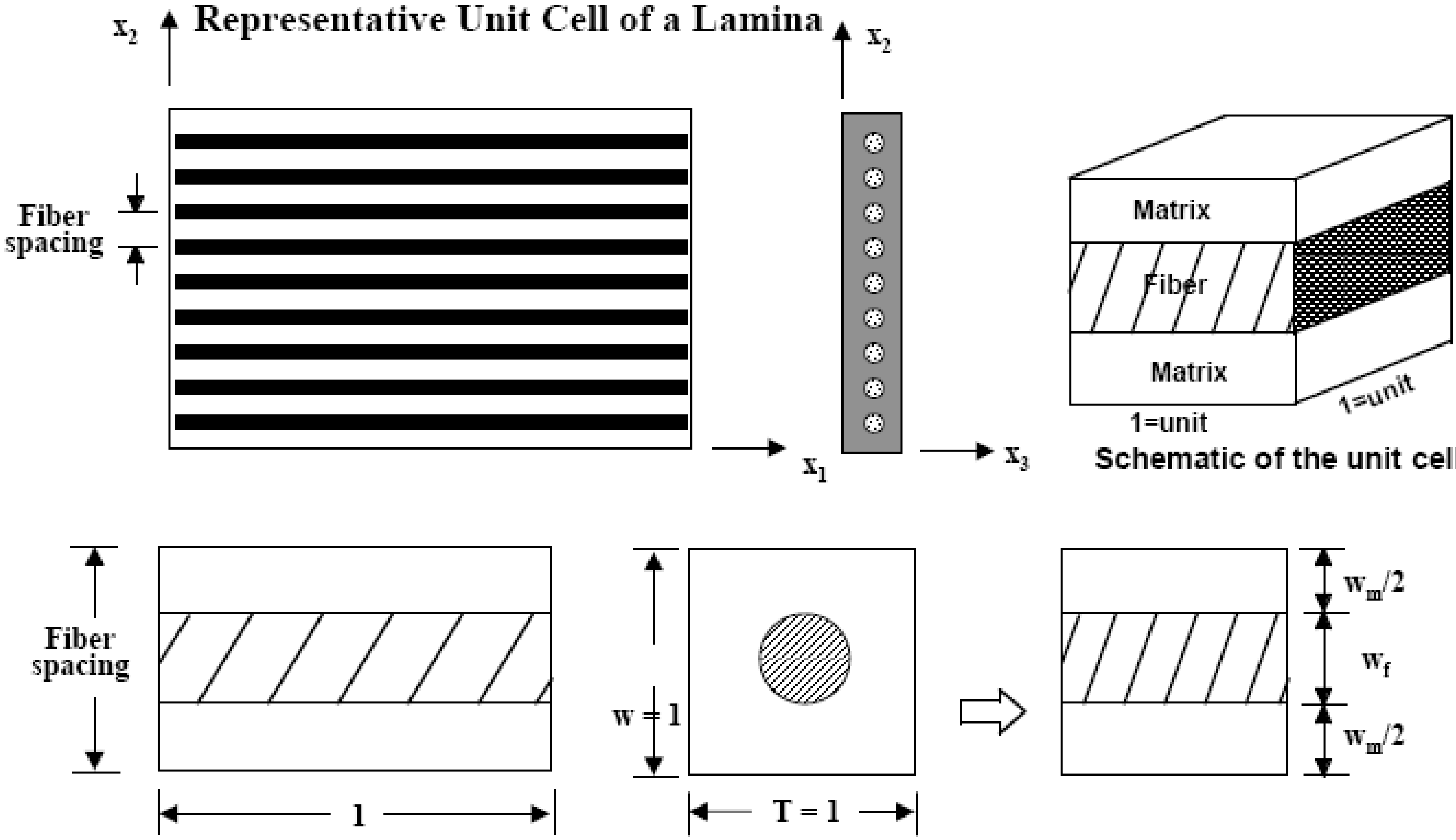
- Longitudinal Modulus - $\mathbf{E_1}$
- Transverse Modulus - $\mathbf{E_2}$
- Shear Modulus - $\mathbf{G_{12}}$
- Poisson's Ratio - $\mathbf{n_{12}}$
- Interlaminar Shear Modulus

Constituent Materials:

Fiber (Graphite, boron, Silicon): E_f, ν_f, G_f, and V_f

Matrix (Resin): E_m, ν_m, G_m V_m

Mechanics of Materials Method



Volume of fiber = A_f x 1 = A_f

Volume of matrix = A_m x 1 = A_m

Volume of composite = V_c = A_c x 1

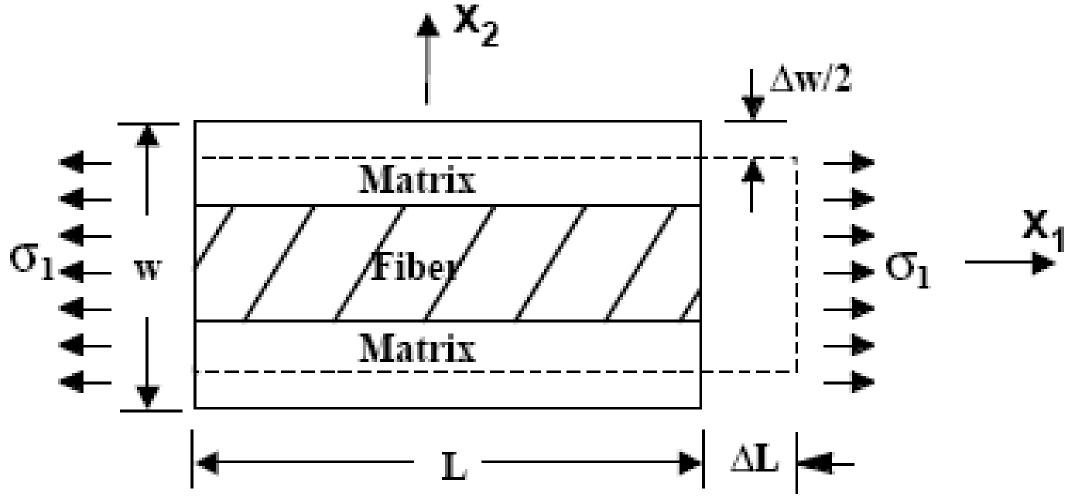
∴ Fiber volume fraction = $A_f/A_c = V_f$

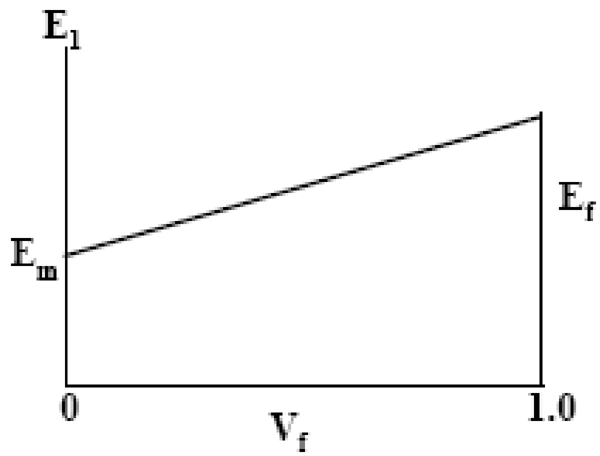
Matrix volume fraction = $A_m/A_c = V_m$

Also $W_f/W = V_f$

and $W_m/W = V_m$

Determination of E_1





Assumption: Axial (x-) strain is same for the lamina, fiber and matrix

∴ **Strain in the composite** $\varepsilon_1 = \dfrac{\Delta L}{L} = \varepsilon_f = \varepsilon_m$

Total force in composite $\sigma_1 A_c = \sigma_f A_f + \sigma_m A_m$

∴ **Stress in the composite** $\sigma_1 = \sigma_f \dfrac{A_f}{A_c} + \sigma_m \dfrac{A_m}{A_c} = \sigma_f V_f + \sigma_m V_m$

$$\varepsilon_1 E_1 = \varepsilon_1 E_f V_f + \varepsilon_1 E_m V_m$$

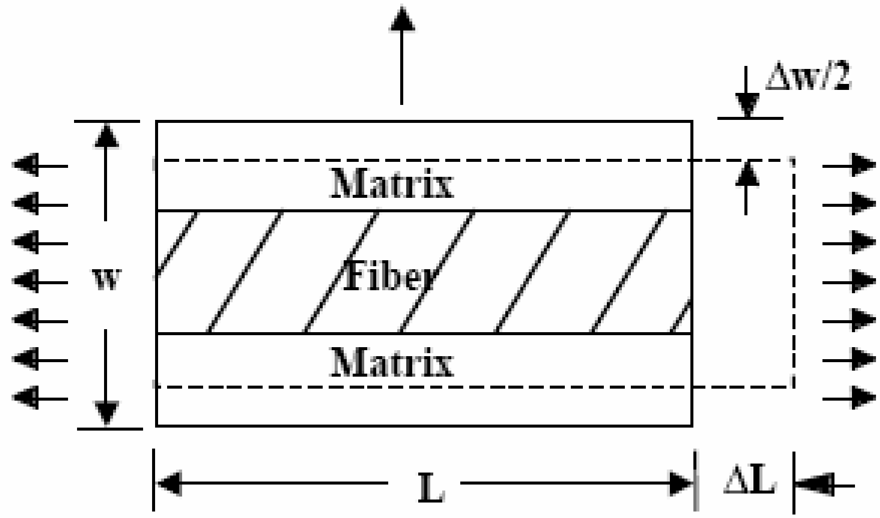
∴ $$E_1 = E_f V_f + E_m V_m$$ ***Rule of mixtures***

Determination of ν_{12}

Lateral strain due to σ_1, $\varepsilon_2 = \frac{\Delta W}{W}$

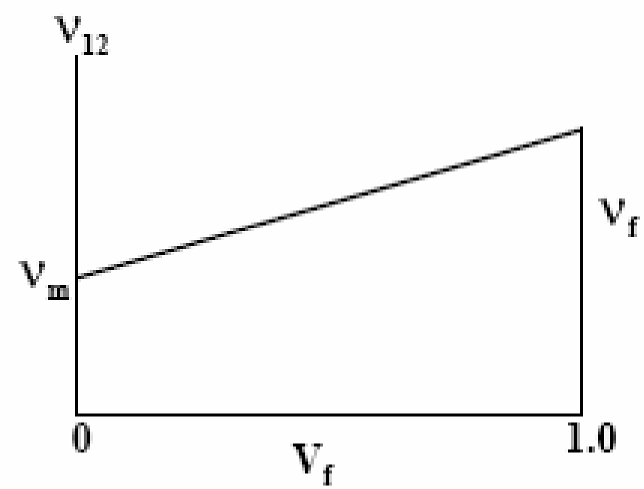
Poisson's ratio (Major) $\nu_{12} = -\frac{\varepsilon_2}{\varepsilon_1}$

$$\varepsilon_2 = \frac{\Delta W}{W} = \frac{-\varepsilon_1 \nu_f W_f - \varepsilon_1 \nu_m W_m}{W}$$



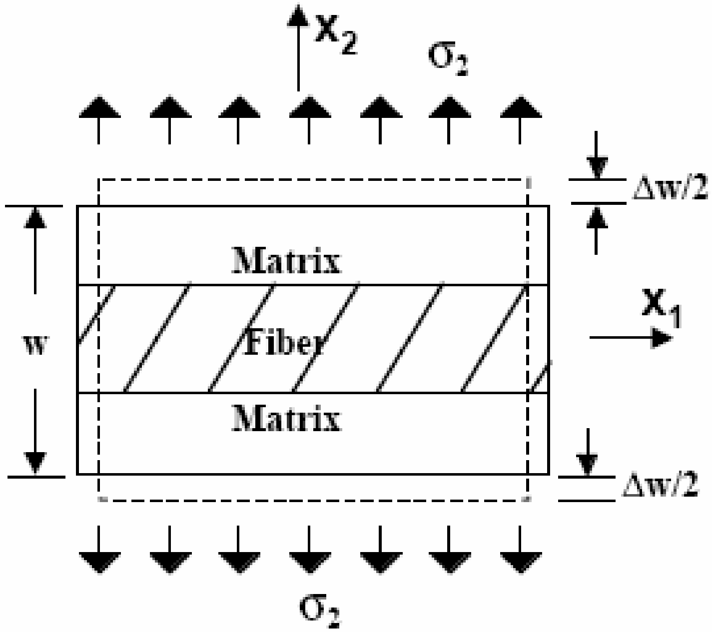
$$\varepsilon_2 = -\varepsilon_1 \nu_f V_f - \varepsilon_1 \nu_m V_m$$

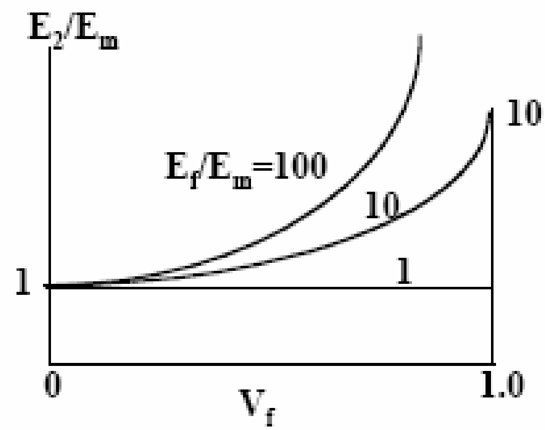
$$\therefore \quad \nu_{12} = -\frac{\varepsilon_2}{\varepsilon_1} = \nu_f V_f + \nu_m V_m$$



$$\nu_{12} = \nu_f V_f + \nu_m V_m$$

Determination of E_2





Assumption: Transverse stress, σ_2, is same in composite, fiber, and the matrix

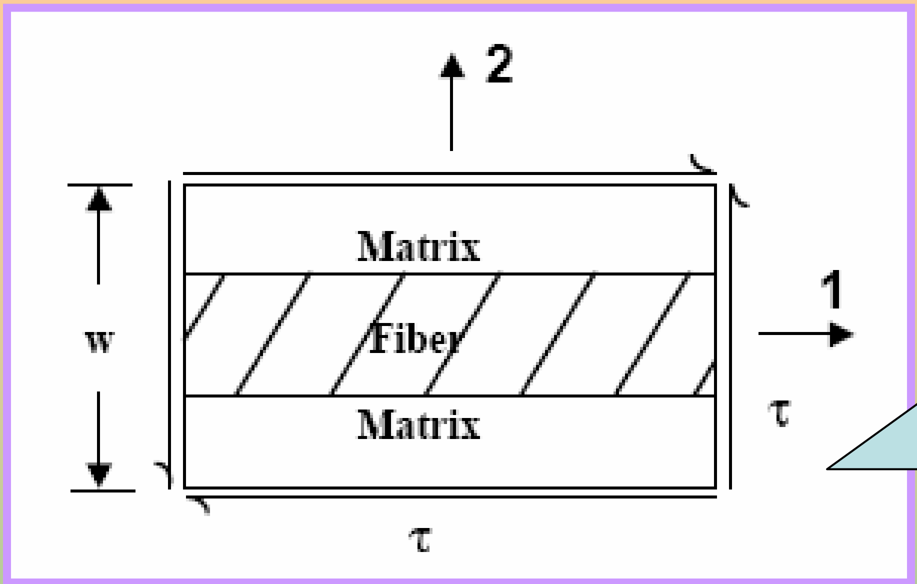
$$\therefore \quad \Delta W = \varepsilon_2 W = \frac{\sigma_2}{E_f} W_f + \frac{\sigma_2}{E_m} W_m$$

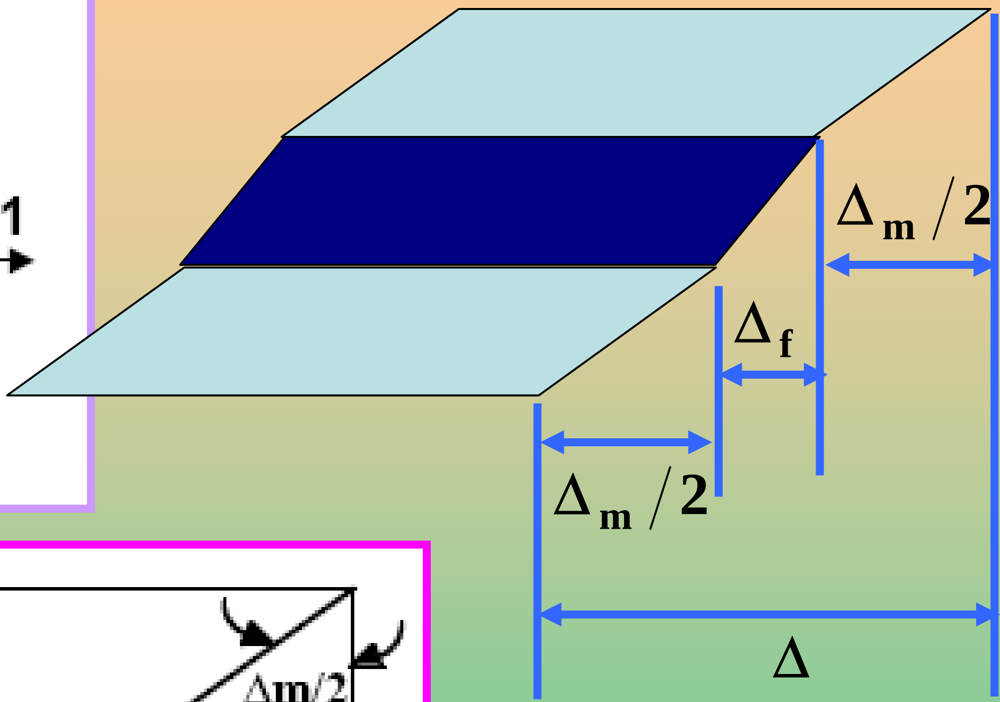
$$or \quad \frac{\sigma_2}{E_2} = \sigma_2 \frac{W_f}{E_f W} + \sigma_2 \frac{W_m}{E_m W}$$

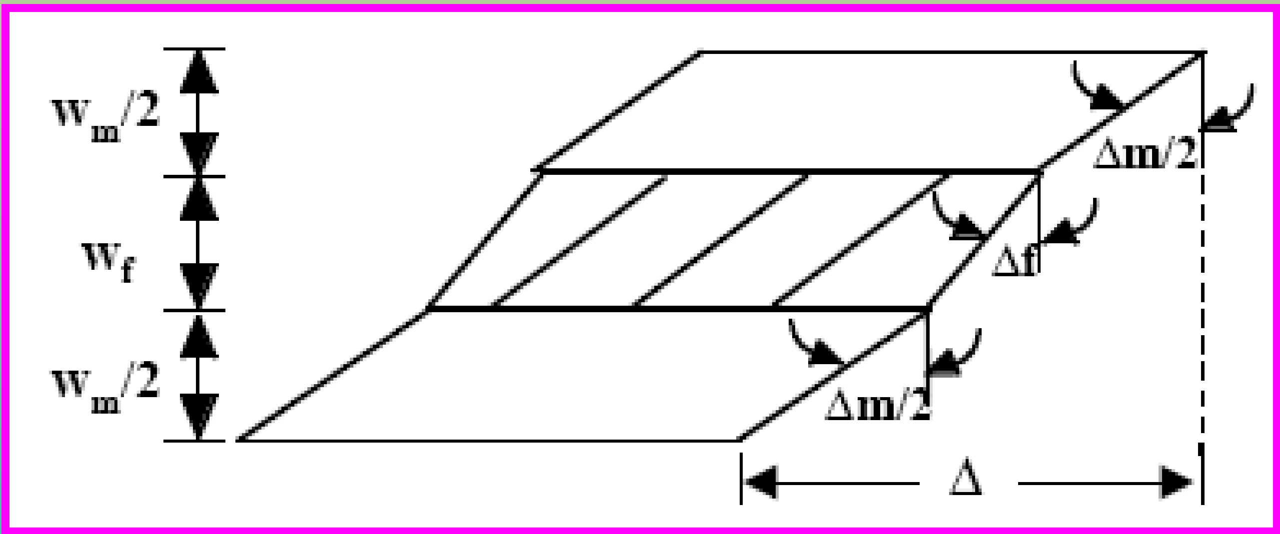
$$\therefore \frac{1}{E_2} = \frac{V_f}{E_f} + \frac{V_m}{E_m}$$

Reciprocal theory

Determination of G_{12}







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Assumption: Shearing stress (τ) in composite, fiber and matrix is same .

Shear strain in Composite: $\gamma = \dfrac{\tau}{G_{12}}$

Shear strain in Matrix: $\gamma_m = \dfrac{\tau}{G_m}$

Shear strain in Fiber $\gamma_f = \dfrac{\tau}{G_f}$

Total shear deformation $\Delta = \gamma W = \dfrac{\tau W}{G_{12}}$

Matrix $\Delta m = W_m \gamma_m$

Fiber $\Delta f = W_f \gamma_f$

$$\Delta = \frac{\tau W}{G_{12}} = \frac{W_m \tau}{G_m} + \frac{W_f \tau}{G_f}$$

$$\boxed{\frac{1}{G_{12}} = \frac{V_m}{G_m} + \frac{V_f}{G_f}}$$

$$G_{12} = \frac{G_{12f} G_m}{V_f G_m + V_m G_{12f}}$$

Inverse Rule of Mixtures

$$\frac{1}{G_{12}} = \frac{v_f}{G_{f12}} + \frac{v_m}{G_m}$$

G_{f12} = *Shear Modulus of fiber in 12-plane*

G_m = *Shear Modulus of matrix*

Inverse Rule of Mixtures

$$G_{12} = \frac{G_m G_{f12}}{v_m G_{f12} + v_f G_m}$$

$$\frac{G_{12}}{G_m} = \frac{1}{v_m + v_f \left(\frac{G_m}{G_{f12}} \right)}$$

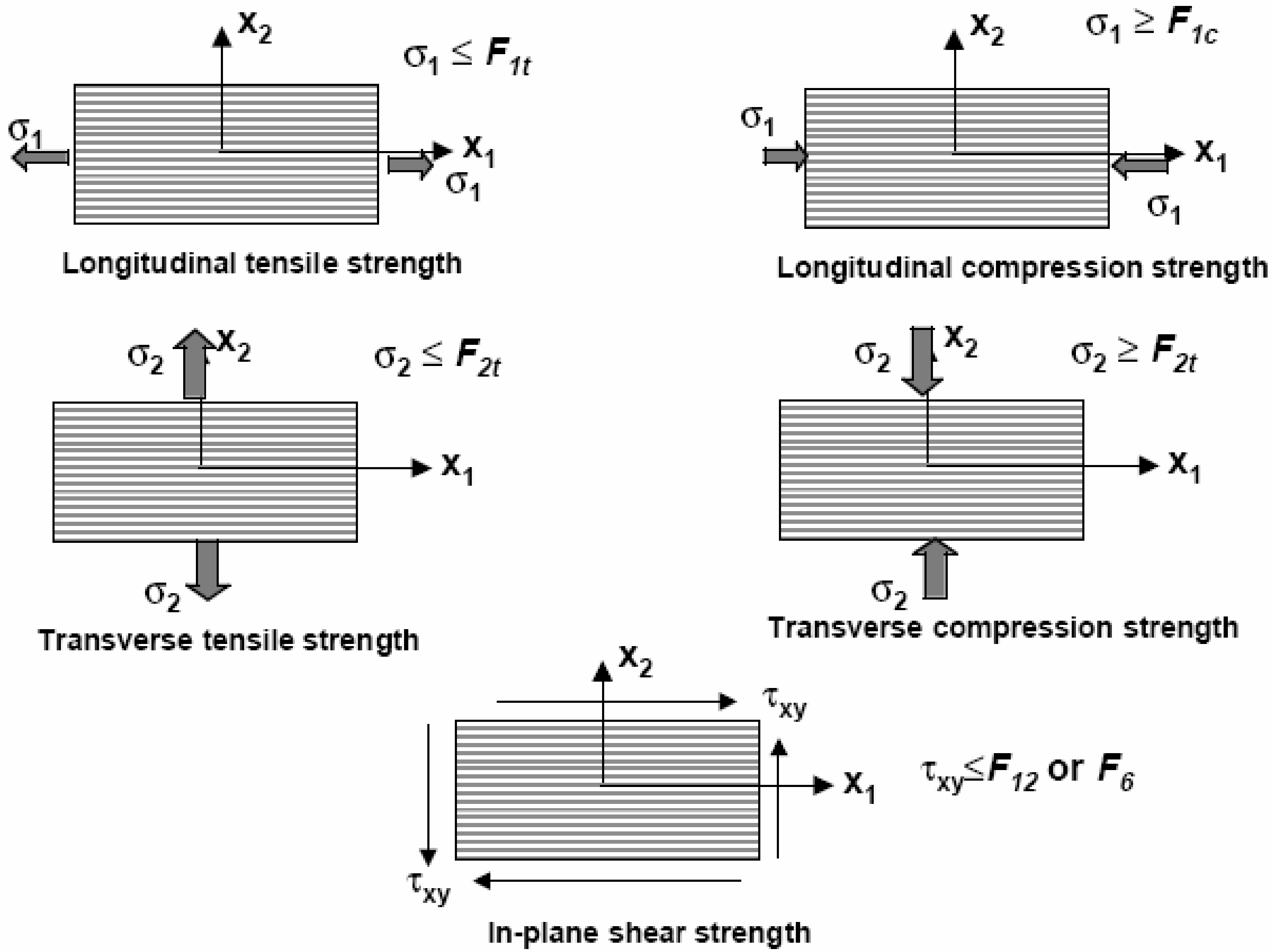
Strength of Materials Approach

Assumptions are made in the strength of materials approach:

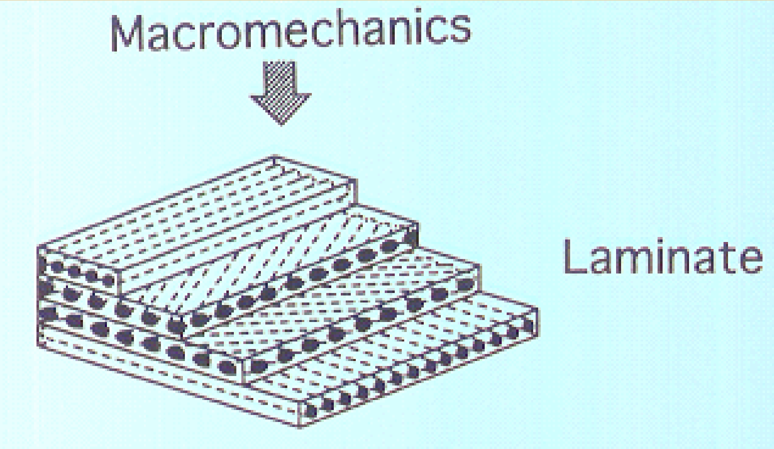
- The bond between fibers and matrix is perfect.
- The elastic moduli, diameters, and space between fibers are uniform.
- The fibers are continuous and parallel.
- The fiber and matrix follow Hooke's law (linearly elastic).
- The fibers possess uniform strength.
- The composites is free of voids.

Micromechanics of Strength Models

Principal Strength Parameters of a Lamina



- Laminate Theory
  - which layers?
  - how many layers?
  - how thick?



>>> Laminate Properties

Laminate Properties

>>> Behaviour Under Loads
(Strains, Stresses,
Curvature, Failure Mode…)

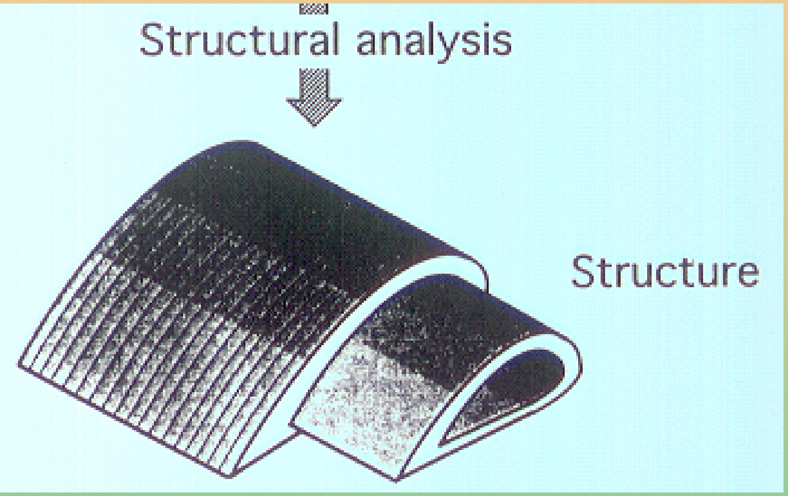
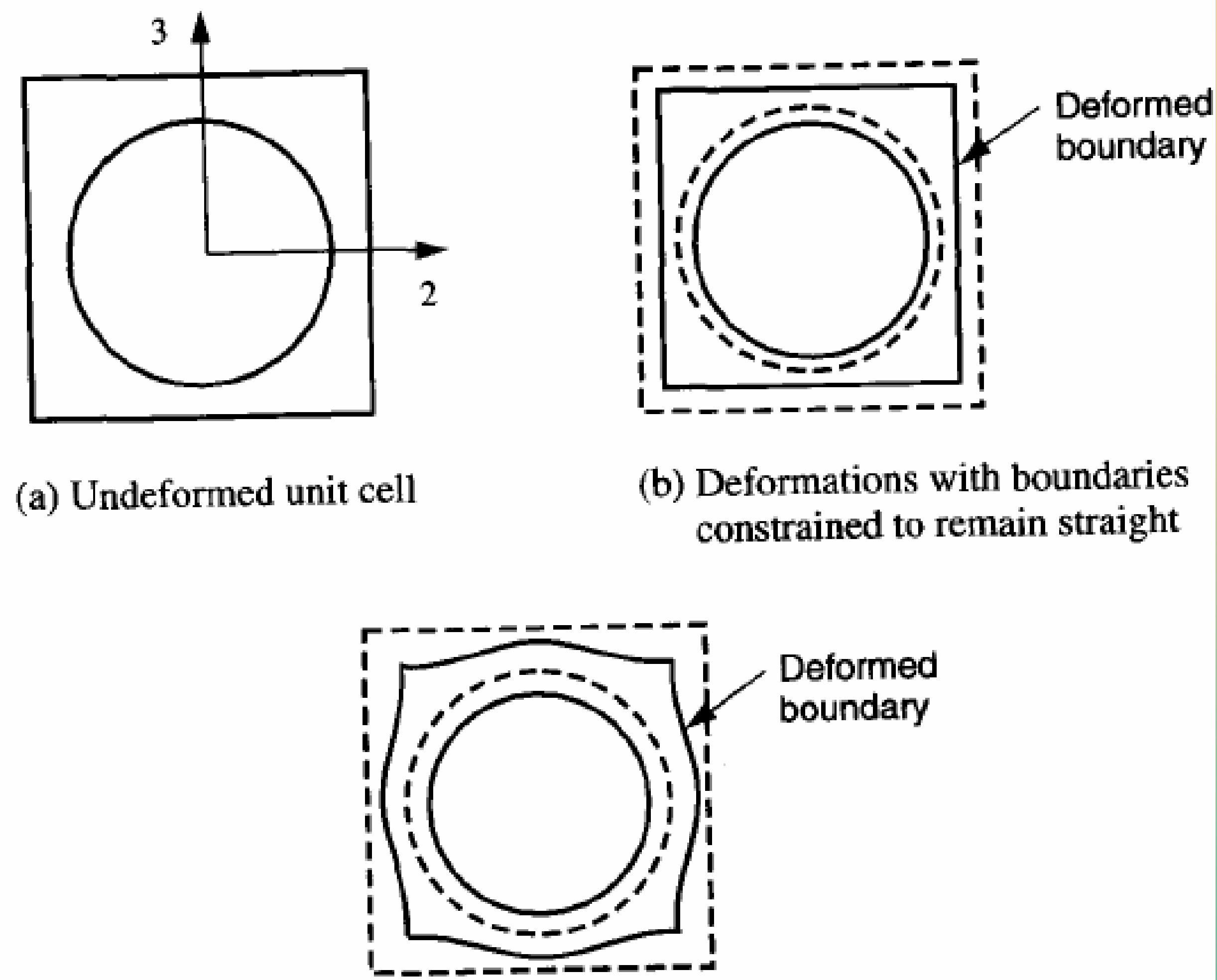


Figure: Cross-sectional deformations of unit cell of graphite-reinforced material due to applied fiber-direction strain



(a) Undeformed unit cell

(b) Deformations with boundaries constrained to remain straight

(c) Deformations with no boundary constraints